In Search of a Unicorn*

Felix Zhiyu Feng[†] Yifan Luo[‡] Beatrice Michaeli[§]

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[†]Felix Zhiyu Feng is at Foster School of Business, University of Washington and can be reached at ffeng@uw.edu.

[‡]Yifan Luo is at Foster School of Business, University of Washington and can be reached at yifanluo@uw.edu.

[§]Beatrice Michaeli is at Anderson School of Management, University of California, Los Angeles and can be reached at beatrice.michaeli@anderson.ucla.edu

In Search of a Unicorn

Abstract: Searching for investment opportunities is one of the fundamental responsibilities of corporate managers. This paper studies a dynamic agency model where investors contract with a manager to find opportunities ("targets") arriving stochastically. The model has two novel features frequently observed in practice: First, investment targets arrive at different levels of quality that are only observable by the manager. Second, once the investment target is chosen, the manager is put in charge of the ensuing operations and can continue to utilize her private information about the target to extract rents. These novel features imply an interaction between adverse selection and moral hazard problems. We find that the optimal contract has a progressively lower threshold for investment over time and leads to overinvestment. Our model generates several empirical predictions regarding the strategies and returns of mergers and acquisitions, hedge fund activism, venture capital investing, and special purpose acquisition companies.

JEL classification: G32, D86, M11

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1 Introduction

Identifying investment opportunities is an important managerial role in modern businesses. CEOs pursue acquisition opportunities to maintain corporate growth. Managers of activist hedge funds seek undervalued firms for intervention. General partners of venture capital funds look for promising startups to invests in. Sponsors of special purpose acquisition companies hunt for the most valuable private firms—the "unicorns"—to merge with. Traditionally, the processes above are studied in a random discovery framework with moral hazard as the main friction. In this framework, the search requires unobservable effort from the manager, who is also privy to the search results (e.g., Green and Taylor, 2016). When the manager discloses the arrival of a search target, only the timing of such disclosure matters, and the manager's role ends.

We study the search for valuable investment opportunities in a dynamic agency model that incorporates two novel features. First, besides the timing of arrival, the *quality* of the investment opportunity also matters. Because the manager is privy to this quality, an additional agency friction—adverse selection—arises and interacts with the conventional moral hazard problem during the search process. Second, the role of the manager does not end with the disclosure of the investment opportunity. These two features are commonly observed in practice where investment opportunities often differ in terms of their potential and managers handle the ensuing operations related to the undertaken investments. For example, CEOs of the acquirers operate the combined firms; managers of activist hedge funds oversee the revamping of the undervalued targets; and venture capitalists and special purpose acquisition companies play critical roles in the growth and development of their investments. In these situations, managers can take advantage of their private information and continue to extract personal benefits after the search has concluded, which in turn affects the investors' return from the search, the type of opportunities they are willing to invest in, and the overall investment efficiency.

Our model has two distinct stages: in the first (search) stage, a group of investors contract with a manager ("she") to find business opportunities ("targets") that arrive randomly over time via a Poisson process only if the manager exerts effort. The arrival and the quality of the target are privately observed by the manager. Investors decide whether to end the search and begin production based on the manager's report about target arrival and quality. In the second (production) stage, the manager generates an output that depends on the target quality. The investors' objective is to design the optimal contract that maximizes their total return net of the manager's compensation and provides incentives for the manager to exert the search effort and to report all private information truthfully in both stages.

Absent agency frictions, the investors' first-best strategy is to invest in the first target that clears a sufficiently high constant bar of quality (threshold) and never abandon the search. The information frictions, however, create moral hazard and adverse selection problems that interact with each other. In the production stage, where productivity is privately observed by the manager, the incentives for truthful reporting are provided in the form of excess compensation, known as the manager's *information rent*. This rent is increasing in the quality of the target, which determines the expected return received by investors from a given investment policy. In the search stage, incentives are provided in the form of promised utility. The investors specify in the contract their investment policy and the reward for reporting the arrival of suitable targets.

As in the first-best, the optimal investment policy when agency frictions are present takes the form of a threshold. Early on in the search stage, the investment threshold is high, and the manager is just indifferent between exerting the search effort and shirking to consume the search resources for private benefits. If the manager does not report the arrival of a target, her promised utility drifts down to provide incentives for her to maintain the search effort. As a result, the optimal cutoff declines, and the manager begins to strictly prefer exerting the effort to conduct the search. The decline continues until all targets regardless of their quality trigger the start of production. If a target is still not found after sufficiently long search, the manager's contract is terminated without pay. Critically, the optimal investment threshold induced by the agency frictions is always below the first-best level. That is, there is *overinvestment*.

We also extend our analysis by incorporating dynamics in the production stage. In this model variation, the target quality determines only the initial productivity, which evolves over time afterward subject to time-varying shocks. The evolution path of productivity is privately observed by the manager and the investors dynamically adjust their production policies based on the manager's reported productivity. The main results remain robust. This extension provides a unified framework to jointly study two important agency frictions—dynamic adverse selection and dynamic moral hazard.

The main results of our model and their comparative statics generate empirically testable predictions regarding mergers and acquisitions (M&A), hedge fund activism (HFA), venture capitalists (VC), and special purpose acquisition companies (SPAC), where evidence for overinvestment abounds.[1] Prior literature (e.g., Shleifer and Vishny, 1997; Franzoni, 2009; Malenko, 2019; Gregor and Michaeli, 2020, 2022) often attributes overinvestment to the manager's empire-building preferences. This paper offers an alternative explanation based on optimal contracting under agency frictions. We expect that overinvestment is correlated with the incentives' power of the acquirer, activist fund managers, VC executives or the SPAC management team. The model also predicts that overinvestment and average return from the above-mentioned activities are positively associated with (i) number of firms, frequency of deals and average returns in these markets; (ii) geographical proximity, executive connections, stock liquidity, analyst coverage, and institutional holdings of the targets. Another prediction is that the returns in these markets should be more dispersed when there are more targets available or when the manager's incentives are low-powered.

2 Related literature

Our paper belongs to the literature studying dynamic contracting models with Poisson jumps (e.g., Biais, Mariotti, Rochet, and Villeneuve, 2010, Hoffmann and Pfeil, 2010, Piskorski and Tchistyi, 2010, DeMarzo, Fishman, He, and Wang, 2012, DeMarzo, Livdan, and Tchistyi, 2013, Myerson, 2015, Sun and Tian, 2018, Rivera, 2020, and Feng, 2021). The most closely-related studies are Green and Taylor (2016), Curello and Sinander (2021), Madsen (2022), and Mayer (2022). These studies assume that through effort, the agent can observe a private signal that is valuable to the principal. The optimal contract provides incentives for the agent to exert the effort to uncover the signal and to report it as soon as it arrives. Our paper

[1] See, e.g, Andrade, Mitchell, and Stafford (2001), Eckbo (2008), Gahng, Ritter, and Zhang (2022), etc.

differs from these studies in two dimensions. First, in addition to the arrival time, the agent (manager in our case) also privately observes the quality of the private information and must be incentivized to truthfully convey both the arrival and the quality to the principal (investors in our case). Second, the contracting relationship does not end with the disclosure of the private information. The value of the information manifests in a production process, during which the manager can continue to utilize her private information to extract rents.

The introduction of these new dimensions creates an adverse selection problem with unknown agent types (i.e., the quality of the target the agent possesses) in addition to the dynamic moral hazard problem. Cvitanić, Wan, and Yang (2013) studies an extension of the dynamic contracting problem in DeMarzo and Sannikov (2006) by assuming that one of the manager's characteristics (such as the private utility she enjoys if she diverts cash flows) is her private information and the investors design a screening contract to infer that information *before* hiring the manager. This turns out to be analytically difficult, as the contract must always keep track of at least two state variables: one for each type of the manager if she accepts the contract. Consequently, the solution in Cvitanić, Wan, and Yang (2013) relies mainly on numerical simulation. In contrast, our model assumes that the adverse selection arises *after* the manager is hired, which allows us to separate adverse selection from moral hazard. Crucially, the screening of the manager's private information can be made relatively independent of the manager's moral hazard problem, which substantially reduces the technical hurdles in deriving the optimal contract. Meanwhile, there is still a meaningful interaction between the two frictions because the design of the screening contract affects the dynamics of the incentives during the search stage. Thus, the model yields different but practically relevant predictions than those in models with either only adverse selection or only moral hazard, which we discuss in Section 6.

Meanwhile, Varas (2018) studies a dynamic model of managerial short-termism in which the manager's private information is binary: the manager can either spend effort and time to discover a value-enhancing good project, or pass off a value-destroying bad project which is always available. The investors, who do not observe the manager's effort nor the quality of the project, use a contract with decreasing compensation as the incentive for effort. However, to prevent short-termism, the contract holds the compensation stationary at some point and switches the incentive to random termination. In the equilibrium, the bad project is never invested, i.e., short-termism never occurs. Our model differs in that there is a continuum of private information: the quality of the investment target observable only to the manager. Thus, the optimal contract provides time-varying incentives both for the manager's search effort and for the truthful reporting of her private information at different point in time. This results in the investors in our model becoming endogenously more "short-termist," as they progressively permit the investment of lower-quality targets over time. Also, while the contractual relationship in Varas (2018) continues after the investment project is chosen, the manager takes no action in that period and cannot influence the project output. In contrast, the manager in our model is also in charge of production from the chosen project and can manipulate project output to extract more rents.

More broadly speaking, our paper is also related to the literature on mechanism design in which the agent can take private actions.[2] For example, Krähmer and Strausz (2011) and Liu and Lu (2018) study the optimal procurement policy in two-period models where the agent is able to influence the principal's screening outcome in the second period through private actions in the first period. The agent makes only one report about some private information and the contract either moves on to the next stage (if the report clears a pre-specified hurdle) or terminates altogether. In comparison, our search stage is fully dynamic and thus features endogenous termination and stochastic transition time to the second stage. The agent is allowed to make repeated reports about the arrival and quality of the targets until either transition or termination occurs. The dynamics also imply that the criterion for transition is type-varying as the result of previous reports. Related, Halac, Kartik, and Liu (2016) studies experiments in a learning model in which the agent's private effort is a necessary (but not sufficient) condition for success. The adverse selection of the underlying success likelihood results in a screening contract with different endogenous deadlines at which the contract is terminated if success has not been achieved. Our model shares the similarities that "success" (the arrival of an investment target) is stochastic and only possible if the manager exerts effort. However, our model differs in that "success" carries the additional information

[2] In addition to the mechanism design studies discussed in detail, see also Sung (2005), Garrett and Pavan (2012, 2015), Gershkov and Perry (2012), Chassang (2013), Gary-Bobo and Trannoy (2015), Gershkov, Li, and Schweinzer (2016), Duggan (2017), Shan (2017), Che, Iossa, and Rey (2021).

about the quality and thus requires different levels of incentives under the optimal screening contract. This additional dimension of the agent's private information also implies that the definition of "success" in our model is time-varying: earlier targets must clear a higher hurdle in order to successfully trigger investment.

In terms of empirical predictions, our paper complements recent literature that also features overinvestment as a result of dynamic agency frictions with different mechanisms, e.g., Bolton, Wang, and Yang (2019) and Ai, Kiku, Li, and Tong (2021) with limited commitment, Gryglewicz, Mayer, and Morellec (2020) with correlated short-run and long-run effort, Szydlowski (2019) with multi-tasking, Gryglewicz and Hartman-Glaser (2020) with real options, and Feng (2022) with persistent effect of moral hazard. All of these studies assume the output process is driven by Brownian shocks and the level of investment can be adjusted constantly and smoothly. Consequently, their models apply more naturally to operational investments such as capital expenditure or research and development expenses. In contrast, our paper considers a setting in which the investment targets arrive via a Poisson process if the manager exerts a sufficient search effort. Our theory is therefore more applicable to lumpy investments such as M&A, HFA, VC and SPAC.

3 Economic Setting

Below we first describe the model ingredients (in Section 3.1) and then discuss some of our assumptions (in Section 3.2).

3.1 Model Description

We consider a group of investors contracting with a manager ("she"). The investors have deep pockets while the manager is protected by limited liability. Both parties are risk-neutral with no discounting, and their outside options are normalized to 0. There are two stages—search and production—and time is continuous.

Search stage. During the search stage, the manager is tasked with finding an investment target (e.g., a promising startup for acquisition or an undervalued firm for activism intervention). The search incurs a cost δ that must be financed by the investors. Each target is characterized by its quality θ, which follows a Pareto distribution with cumulative distribution function $F(\theta) = 1 - \left(\frac{\theta_{\min}^{\kappa}}{\theta^{\kappa}}\right)$ and probability density function $f(\theta) = \frac{\kappa\theta_{\min}^{\kappa}}{\theta^{\kappa+1}}$. The distribution scale parameter is positive, $\theta_{\min} > 0$, and the shape parameter is sufficiently large, $\kappa > 2$.

There are two agency frictions during this stage. First, the manager controls her search effort, which is unobservable to the investors. If the manager exerts the effort, the targets arrive via a Poisson jump process N_t with intensity λ. If the manager shirks, she obtains a private benefit ρ, and no target arrives. Here ρ represents the perks and benefits from actions that are enjoyable to the manager personally but do not contribute to the discovery of investment targets, such as excessive traveling, spending the firm/fund's resources to build personal reputation or network, and hiring (unqualified) friends and family members. We assume $\rho < \delta$, so shirking is socially inefficient. The second friction is that only the manager observes the arrival of targets and their quality θ. The investors decide whether to invest in the target based on information reported by the manager. The length of this stage is endogenous: the search ends either when the investors invest and move on to the next (production) stage, or when the contract is terminated.

Production stage. During the production stage, the manager generates output from the target chosen by the investors during the previous stage. The production technology is $y(\theta, e) = \theta e$, where e is the manager's unobservable production effort exerted at a quadratic personal cost $h(e) = e^2/2$. Similar to the first stage, the target quality is privately observed only by the manager. The output y, however, is observable by the investors. This implies that the main agency friction in the production stage is adverse selection, because a manager with a low-quality target can always mimic the output of a manager with a high-quality target (or vice versa) by exerting higher (lower) effort. The unobservable effort in this stage only provides cover for the manager so that the true quality of the target cannot be inferred with certainty based on the observable output. In contrast, the agency frictions in the first stage involve both moral hazard and adverse selection, because the manager can shirk and/or lie about the target arrival and/or quality.

Contract. A contract $\mathcal{C}$ between the investors and the manager consists of the investors' investment and production policies, and the associated compensation to the manager. During the search stage, the contract specifies the set of targets that will be invested in, the reward for the manager for announcing the arrival of the target, and the condition under which the contract is terminated. During the production stage, the contract specifies how the manager will be compensated based on the observed production. A contract is incentive-compatible if the manager finds it optimal to always exert the desired search effort and announce her private information truthfully.

3.2 Discussion of Assumptions

We now discuss in detail the roles of several simplifying assumptions of the model:

1. *Discounting.* Our model assumes that there is no discounting for the investors and the manager. This assumption is common among models in which the arrival of information follows a Poisson process (e.g., Green and Taylor, 2016; Mayer, 2022). Discounting implies investors prefer earlier resolution, which distorts their investment threshold downward. Except for this result, discounting usually adds little economic intuition in these types of models but a substantial degree of algebraic complexity.

2. *Continuous time.* The assumption that time is continuous allows for more elegant analysis. A setting with discrete time leads to several analytical complications (such as the need for randomized termination) but does not qualitatively change our results.

3. *Pareto distribution.* We assume that the target quality θ follows a Pareto distribution because of its broad applications in economics and its analytical convenience. In particular, this distribution belongs to the power-law family and is descriptive of many economic variables and activities in practice (e.g, Gabaix and Landier, 2008). Furthermore, the distribution has two analytical advantages. First, the inverse hazard rate $[1 - F(\theta)]/f(\theta) = \theta/\kappa$ is a linear function of θ. This significantly simplifies the proof of Proposition 1 and all subsequent analysis. Second, a Pareto distribution truncated from below at an arbitrary point $x > \theta_{\min}$ is also a Pareto distribution with the same shape parameter and the new scale parameter x. This is vital for tractability. The requirement $\kappa > 2$ is technical and implies a sufficiently thin right tail of the distribution

to ensure a finite variance of θ and a finite solution to the first-best (see footnote 4 for more details)

4. *Production technology.* Our model assumes that the production technology is $y(e, \theta) = e\theta$. This implies that managerial effort and project quality are perfect complements and achieves two useful simplifications. First, in equilibrium, production effort is never shut down regardless of target quality.[3] Second, the manager cannot generate output without a target. Consequently, while the manager can misreport the quality of the arriving targets, she cannot fabricate their existence. This assumption is intuitive in the context of M&A, HFA, VC, and SPAC, because the success of the merger/acquisition or the intervention require cooperation from the targets. While managers of the acquiring firm or the activist/VC fund can exaggerate the true value of the targets to their investors, they are usually not able to create phony targets.

In summary, the assumptions discussed above facilitate tractability and are *not* crucial for our results. The predictions of the model remain qualitatively unchanged if we relax any of these assumptions (e.g., switch to discrete time, allow discounting, use an alternative distribution for θ and/or production function for y).

4 First Best Benchmark

If all information is public, the first-best effort and output in the production stage will maximize the social surplus from production. i.e.,

$$\max_{e} \; y - h(e) = \theta e - h(e). \tag{1}$$

The first-best effort and output,

$$e^{FB} = \theta, \tag{2}$$

$$y^{FB} = \theta^2, \tag{3}$$

[3] In contrast, if the production technology is linear, e.g., $y = e + \theta$, then under the optimal contract, effort may be shut down if target quality θ is sufficiently high.

are both increasing functions in θ. The manager is only compensated for her cost of effort, and the payoff to the investors is $V_2^{FB}(\theta) = \theta^2/2$.

The search stage under the first-best scenario represents a standard bandit problem. Let Θ^{FB} denote the set of targets that will be invested in and V_1^{FB} denote the investors' expected value at the outset of the search stage.

Lemma 1 *Under the first-best scenario, $\Theta^{FB} = \{\theta : \theta \geq x^{FB}\}$, where*

$$x^{FB} = \left[\frac{\lambda \theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right]^{\frac{1}{\kappa-2}}. \tag{4}$$

The investors' first best expected payoff at the beginning of the search stage is

$$V_1^{FB}(x^{FB}) = \frac{\kappa \left(x^{FB}\right)^2}{2(\kappa-2)} - \frac{\delta \left(x^{FB}\right)^{\kappa}}{\lambda \theta_{\min}^{\kappa}}. \tag{5}$$

The optimal strategy of the investors under the first-best scenario is to finance the search with a constant *minimal* (cutoff) quality x^{FB}.[4] The manager is required to always exert the search effort and thus receives no private benefit from shirking. Because x^{FB} also determines the expected duration of the search, Lemma 1 suggests that investors on average wait longer in the first-best scenario if there are many opportunities on the market (higher λ) or if the search cost (δ) is low.

5 Optimal Contract Under Asymmetric Information

We now analyze the optimal contract when the efforts and target quality are manager's private information. We solve the model by backward induction.

5.1 Production Stage

The main friction faced by investors in the production stage is adverse selection. For ease of exposition, imagine the following reduced problem without the search stage: the manager is endowed with a target of quality θ that is unobservable by investors. The manager has

[4] Equations (4) and (5) illustrate the need to assume $\kappa > 2$. Otherwise, x^{FB} and V_1^{FB} are not well-defined.

reservation utility W_{τ^-}, which in this reduced problem is given (but in the full-fledged problem represents the utility carried over from the search stage). The investors must design a screening contract that solicits truthful reporting of θ while maximizing their payoff, which is the output net of the manager's compensation.

Based on the revelation principle, we can without loss of generality consider the screening contract as a direct mechanism: the manager reports her type $\widehat{\theta}$ and receives an output target $y(\widehat{\theta})$ and associated compensation $w(\widehat{\theta})$ if and only if the output is produced.[5] Given the contract, the manager's objective is to maximize her compensation net of her (production) effort cost:

$$R(\theta) = \max_{\widehat{\theta}} \quad w(\widehat{\theta}) - h(e) \tag{6}$$

subject to the constraint

$$e = y(\widehat{\theta})/\theta, \tag{7}$$

because she needs to exert the necessary effort to produce the required level of output $y(\widehat{\theta})$ in order to receive compensation. This constraint illustrates that although effort is unobservable in the production stage, the underlying agency friction is only adverse selection: effort merely provides a cover for the manager's report $\widehat{\theta}$ so that her true type θ always remains hidden.

The contract is incentive compatible if and only if

$$\theta = \arg\max_{\widehat{\theta}} \quad w(\widehat{\theta}) - h\left(\frac{y(\widehat{\theta})}{\theta}\right). \tag{8}$$

When (8) is satisfied, $R(\theta)$ is known as the manager's *information rent*: the amount of utility (in excess of her reservation utility) that she must receive in order to truthfully reveal her

[5]Such contract is feasible because, given θ, there is no uncertainty or noise in the production technology. Note that this contract can be alternatively written in a standard "pay-for-performance" form: a function $w(y)$, under which the manager is free to produce any level of output y and receive the corresponding wage $w(y)$, and no reporting is necessary. These two formulations are equivalent because in the equilibrium, both the output target $y(\widehat{\theta})$ and the wage $w(\widehat{\theta})$ are strictly increasing in the manager's reported type $\widehat{\theta}$, thus creating a one-to-one mapping between output and wage. Remark 1 below demonstrates such equivalence in detail. Following the accepted standard in the literature, we consider the direct mechanism that involves reporting because of its transparency in demonstrating the incentive power of the contract.

private information.

Because the investors do not observe θ, their objective is to maximize the *expected* output net of the manager's compensation. The expectation is taken over the distribution of θ, which in equilibrium is the result of the investment policy. Similar to the case of the first-best, we can show that despite the asymmetric information, the optimal investment strategy is still one characterized by a threshold, denoted by x:

Lemma 2 *The optimal investment policy is a threshold one: $\Theta = \{\theta : \theta \geq x\}$, where x is the investor's choice.*

Intuitively, if a target of quality $\bar{\theta}$ will trigger investment under some incentive-compatible contract, the investors can always induce truthful reporting and thus invest in all targets with better quality (i.e., $\theta > \bar{\theta}$) by setting the output target to be $y(\bar{\theta})$ and the wage to be $w(\bar{\theta})$ for those targets.[6] Therefore, the investors' expected net return must be at least weakly increasing in target quality, and excluding high-quality targets is sub-optimal. As a result, the investor's maximal payoff from the screening contract can be written as $V_2(x) - W_{\tau^-}$, where $V_2(x)$ solves

$$V_2(x) = \max_{y(\widehat{\theta}),w(\widehat{\theta})} \int_x^{+\infty} [y(\theta) - w(\theta)] \left(\frac{\kappa x^\kappa}{\theta^{\kappa+1}} \right) d\theta \tag{9}$$

$$= \max_{y(\widehat{\theta}),w(\widehat{\theta})} \int_x^{+\infty} [y(\theta) - h(e(\theta)) - R(\theta)] \left(\frac{\kappa x^\kappa}{\theta^{\kappa+1}} \right) d\theta \tag{10}$$

subject to the IC condition (8).[7] In other words, given the investment policy x, $V_2(x)$ captures the investors' expected payoff from production under the incentive-compatible screening contract with optimally designed output-compensation combinations.

Deriving $R(\theta)$ and $V_2(x)$ represents a static mechanism design problem of which the solution is as follows:

Proposition 1 *Let $\gamma \equiv \frac{\kappa}{\kappa+2} < 1$. Given any investment threshold $x \geq \theta_{\min}$ set in the search stage, the optimal contract in the production stage has the following properties:*

[6] This is incentive compatible because it requires less effort from the manager to produce $y(\bar{\theta})$ when $\theta > \bar{\theta}$.

[7] The term $\frac{\kappa x^\kappa}{\theta^{\kappa+1}}$ represents the distribution of θ given the investment threshold x.

– *The manager's information rent from a target of quality $\theta \geq x$ is given by*

$$R(\theta) = \frac{\gamma^2}{2}(\theta^2 - x^2); \tag{11}$$

– *The investor's expected payoff from production is given by*

$$V_2(x) = \frac{\gamma\kappa}{2(\kappa - 2)}x^2; \tag{12}$$

– *The optimal output is $y^* = \gamma\theta^2$ and the production effort is $e^* = \gamma\theta$.*

The manager's information rent is a quadratic function of target quality θ and the investor's expected payoff is a quadratic function of the investment threshold x. Compared with the first-best level of effort e^{FB} in (2) and the first-best level of output y^{FB} in (3), adverse selection distorts both the optimal effort e^* and output y^* downward by a constant fraction: $1 - \gamma = 2/(\kappa + 2)$. Intuitively, a higher κ corresponds to a lower variance in θ and therefore, less information asymmetry.

Given x, (11) implies that the conditional expectation of the information rent the manager can receive is

$$U(x) \equiv \mathrm{E}\left[R(\theta) | \theta \geq x\right] = \int_x^{+\infty} \frac{\gamma^2}{2}\left(\theta^2 - x^2\right)\left(\frac{\kappa x^\kappa}{\theta^{\kappa+1}}\right) d\theta = \frac{\gamma^2 x^2}{\kappa - 2}. \tag{13}$$

The closed-form expression for $U(x)$ and $V_2(x)$ greatly simplify the design of the optimal contract in the search stage in the next section.

Remark 1 *The optimal screening contract in Proposition 1 can be implemented via a simple output sharing rule*

$$w(y) = \gamma(y - \gamma x^2) + \frac{\gamma^2 x^2}{2} \tag{14}$$

Under this rule, if the manager produces a minimal amount of output γx^2, she receives a basic wage $\frac{\gamma^2 x^2}{2}$ which exactly offsets her effort cost. Then, for every additional unit of output the manager produces, she receives γ fraction of that as her compensation. This simple *output sharing rule represents an indirect mechanism, under which the manager does not need to report her type and can freely produce any level of output she desires. In comparison, Proposition 1 is derived based on a direct mechanism, under which the manager reports* $\widehat{\theta}$ *and receives an output target* $y(\widehat{\theta})$ *and the corresponding wage* $w(\widehat{\theta})$*. However, these two mechanism are equivalent (as an expected result of the revelation principle), because they are both incentive compatible and deliver the exact same managerial rent* $R(\theta)$ *and the same expected payoff to the investors. Thus, our choice of formulating the solution to the adverse selection problem in the production stage as a direct mechanism is without loss of generality.*

5.2 Search Stage

During this stage, the investors face a moral hazard and an adverse selection problems: incentivize the search effort and procure truthful and timely report of the quality of the arriving targets. While the interaction of the two problems can impose substantial analytical challenges in a general model, our setting allows us to tackle the problems sequentially. In particular, Proposition 1 shows that the solution to adverse selection requires giving the manager her information rent $R(\theta)$ for each target in which the company invests. Consequently, the design of the optimal contract in the search stage can be simplified to focus only on the incentives for the search effort.

Similar to standard agency models with a sole moral hazard problem (especially those also set in continuous-time such as DeMarzo and Sannikov, 2006; Biais, Mariotti, Plantin, and Rochet, 2007; and Sannikov, 2008), incentives for search effort are provided in the form of promised future compensation to the manager. Specifically, let τ denote the stopping time either because of transition to production stage or contract termination, $\{a_t\}_{t\in[0,\tau]} \in \{0,1\}$ denote the agent's search effort, and $\{C_t\}_{t\in[0,\tau]}$ denote the compensation to the agent. The contract can be characterized using the agent's continuation utility W_t, defined as

$$W_t = \mathrm{E}\left[\int_t^\tau \rho(1-a_s)ds + \int_t^\tau dC_s + W_\tau\right]. \tag{15}$$

The first term inside the integral is the agent's private benefit if she shirks (i.e., $a_s = 0$). The last term represents her terminal compensation.

Meanwhile, the contracting space can be simplified as follows:

Lemma 3 *The optimal contract always implements no shirking (i.e., $a_t = 1$) during the search stage. The manager is paid if and only if the contract moves to the production stage and the manager produces the required output (i.e., $dC_t = 0$ for all $t < \tau$ and $W_\tau = 0$ if the contract is terminated without production).*

The first result holds because the search requires a cost δ but shirking generates a benefit $\rho < \delta$ to the manager. Therefore, any contract that involves shirking can be strictly improved by discouraging shirking through compensating the manager for her lost shirking benefit. The second result arises because all players are equally patient, so any intermediate compensation can always be delayed at no cost. Given that the production stage is static without noise or risk, it is without loss of generality to accrue all payments until the output is produced.

With the contracting space simplified, the following proposition characterizes the dynamics of the manager's continuation utility and the IC condition in the search stage:

Proposition 2 *At any time t, the investor's optimal investment policy is a threshold one: $\Theta_t = \{\theta : \theta_t \geq x_t\}$ where $\{x_t\}_{t>0}$ is the investor's choice. Given x_t, the manager's continuation utility W_t evolves according to*

$$dW_t = U(x_t)\left[dN_t - \lambda\left(\frac{\theta_{\min}}{x_t}\right)^{\kappa} dt\right]. \tag{16}$$

The manager exerts the search effort if and only if

$$\left[\lambda\left(\frac{\theta_{\min}}{x_t}\right)^{\kappa}\right] U(x_t) \geq \rho. \tag{17}$$

The contract is terminated if $W_t = 0$.

Proposition 2 entails three results. First, similar to standard models with Poisson search, W_t drifts down if no target arrives. If a target with sufficiently high quality arrives, there is an upward jump in W_t, and the firm moves on to the production stage.

Second, and different from standard models in which the role of the manager ends with the arrival of a search result, the existence of a production stage in our model implies that, given the investment policy x_t, $U(x_t)$ is the manager's expected utility reward (i.e., the expected size of the upward jump in W_t) if a suitable investment target arrives. Here a target is *suitable* if its quality clears the threshold of investment, i.e., $\theta_t > x_t$, which happens at the rate of $\lambda \left(\frac{\theta_{\min}}{x_t}\right)^{\kappa}$. Consequently, the manager faces a tradeoff between working and shirking: shirking yields flow benefits ρdt. However, because no target arrives while she shirks, her continuation utility drifts down at the rate of $U(x_t)\left[\lambda \left(\frac{\theta_{\min}}{x_t}\right)^{\kappa}\right] dt$. The manager prefers not to shirk if the above-mentioned cost exceeds the benefit, which is captured by the IC condition (17).

Third, given that W_t drifts downward in the absence of a suitable target, the contract terminates and the manager receives no payment if she does not find a target after a sufficiently long time has passed. This is because, given the production technology, the manager can only produce something out of an actual target. If not, the manager with a very low W_t may find it optimal to falsely announce the arrival of a suitable target and produce the required output using only her effort. The optimal contract in that case generally involves random termination (as in Green and Taylor, 2016 and Varas, 2018).

Investors have two controls when designing the optimal contract: the investment threshold x_t, and the terminal compensation W_τ. Lemma 3 implies that their expected payoff at any time $t \in [0, \tau]$ under the optimal contract, denoted as $V_{1,t}$, solves

$$V_{1,t} = \mathrm{E}\left[\int_t^\tau -\delta ds + y_\tau - W_\tau\right], \tag{18}$$

subject to the IC constraints (8) and (17). It holds that $y_\tau = y$ if production takes place, and $y_\tau = 0$ if the contract is terminated without production. In other words, the investors pay for the search cost δ. They retain the production output y if investment is ever made, but have to make terminal compensation W_τ to the manager. The analysis so far have pinned down the optimal, incentive compatible terminal compensation: if a suitable target can be found, the manager receives an extra reward, which is $U(x)$ in expectation. Otherwise, she receives no payment if the contract terminates without production. Thus, the ensuing analysis focuses on characterizing the optimal investment policy x_t. Proposition 2 implies that the investor's payoff under the optimal contract can be summarized as a function of the manager's continuation utility, or $V_1(W)$, which solves the following Hamilton-Jacobi-Bellman (HJB) equation with x being the only control variable:

$$0 = \max_x \quad -\delta - \lambda \left(\frac{\theta_{\min}}{x}\right)^\kappa U(x)V_1'(W) + \lambda \left(\frac{\theta_{\min}}{x}\right)^\kappa [V_2(x) - W - V_1(W)] \tag{19}$$

subject to the IC constraint (17). The first term represents the search cost. The second term stems from the drift of dW_t, and the third term represents the change in the investor's payoff if a suitable target is found and the contract moves into production.[8]

Rearranging terms, the HJB equation can be conveniently written as

$$V_1(W) = \max_x \quad V_2(x) - W - U(x)V_1'(W) - \frac{\delta}{\lambda}\left(\frac{x}{\theta_{\min}}\right)^\kappa. \tag{20}$$

The tradeoff faced by investors when setting the optimal investment threshold is as follows: A higher x yields a higher expected payoff once a suitable target arrives: $V_2'(x) > 0$, as seen in (12). The cost, however, is two-fold. First, targets with high quality arrive at a lower rate which leads to higher search cost in expectation (the last term in 20). Second, once a target arrives, the manager is given a higher reward to truthfully reveal the target quality: $U'(x) > 0$, as seen in (13). This higher reward must be accompanied by a faster decline of W to maintain W as a martingale (16), which increases the likelihood of contract termination.

If the IC constraint (17) is slack, the optimal choice of x can be obtained by the first-order condition:

$$V_2'(x) - U'(x)V_1'(W) - \left(\frac{d}{dx}\right)\left[\frac{\delta}{\lambda}\left(\frac{x}{\theta_{\min}}\right)^\kappa\right] = 0 \tag{21}$$

The first term is positive (higher expected payoff from production) and the third term is negative (higher search cost). The middle term captures the marginal continuation utility and its sign depends on W. When W is large, $V_1'(W) < 0$, because the promised compensation to the manager lowers the investors' payoff if a suitable target arrives and the

[8]Conditional on moving into production, the investors' final payoff is the output y net of the wage $w(\theta)$ paid for production and the manager's residual utility W carried over from the search stage. The former can be further divided into the compensation for the manager's production effort $h(e)$ and her information rent $R(\theta)$, all embedded in the definition of $V_2(x)$ in equation (10). Put differently, the investors' final payoff can be written as $y - R - h(e) - W$, where the first three terms are captured (in expectation) by V_2.

search ends. When W is small, $V_1'(W) > 0$, because the primary concern for the investors is the likelihood of contract termination, which increases as W declines. Substituting $V_2(x)$ from (12) and $U(x)$ from (13) into (21) yields the optimal investment policy when the IC constraint is slack:

$$x(W) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(W)\right)\left(\frac{\lambda\gamma\theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}} \tag{22}$$

The optimal investment threshold is increasing in the manager's continuation utility, $x'(W) > 0$.[9] Because W_t drifts down over time, the optimal investment strategy is to adopt a progressively lower threshold for the quality of the arriving targets worth investing in.

The left-hand-side of the IC condition (17) is decreasing in x, because the arrival rate of high-quality targets decreases faster than the manager's expected rent from those targets. Therefore, there may exist $\overline{W}$ such that the IC condition is binding when $W \geq \overline{W}$. In that region, $x(W)$ is given by the IC condition, or

$$x(W) = \bar{x} \equiv \left[\frac{\lambda\gamma^2\theta_{\min}^{\kappa}}{\rho(\kappa-2)}\right]^{\frac{1}{\kappa-2}}, \quad \text{if } W \geq \overline{W} \tag{23}$$

In other words, the optimal investment threshold x is constant for sufficiently high level of W. Setting $x = \bar{x}$ in (22) implies that $\overline{W}$ solves

$$V_1'(\overline{W}) = \frac{\kappa}{2}\left(\frac{1}{\gamma} - \frac{\delta}{\rho}\right) \tag{24}$$

Meanwhile, $\theta_{\min}$ represents the lowest investment threshold that the investors can set. Substituting $x = \theta_{\min}$ into (22) implies that $x(W) = \theta_{\min}$ for all $W \leq \underline{W}$, where $\underline{W}$ solves

$$V_1'(\underline{W}) = \left(1 - \frac{\delta(\kappa-2)}{\gamma\lambda\theta_{\min}^2}\right)\frac{\kappa}{2\gamma} \tag{25}$$

That is, when W is sufficiently low, the optimal policy is to invest in the next target that arrives, regardless of its quality. This maximizes the probability that the contract moves to

[9] Technically speaking, this is because $V_1(W)$ is a concave function, i.e., $V_1''(W) < 0$, which is a standard feature of dynamic contracting models and can be visualized in Figure 1.

the production stage before it is terminated.[10]

Finally, we impose the following parameter assumption to maximize the economic values of the $\overline{W}$ and $\underline{W}$ derived above:

Assumption 1 *The parameters $\lambda, \theta_{\min}, \kappa, \delta, \rho$ satisfy the following conditions:*

$$\lambda\theta_{\min}^2 > \frac{\rho(\kappa-2)}{\gamma^2}; \tag{26}$$

$$\rho > \gamma\delta. \tag{27}$$

The first condition ensures that $\underline{W} < \overline{W}$, so they both exist under the optimal contract. Intuitively, this condition implies that the search is valuable to investors in terms of arrival rate and target quality. Therefore, the investors find it worthwhile to wait for a target with sufficient quality as long as W is not too low and termination is not imminent. If this condition is not satisfied, investors may find it optimal to minimize the waiting time by investing in the first target regardless of its quality.

The second condition ensures that $V_1'(\overline{W}) > 0$. Intuitively, $V_1'(W) < 0$ if W is sufficiently high, because a larger promised utility to the manager diminishes the investors' payoff when a target is found. However, because W drifts downward in the absence of a suitable target, investors can set the manager's initial continuation utility at the outset of the search stage to be $W^* = \arg\max_W V_1(W)$, or $V_1'(W^*) = 0$. Once the search begins, W drifts down into the region in which $V_1'(W) > 0$ until either a suitable target is found or the contract is terminated. Condition (27) therefore ensures that $\overline{W}$ is on the equilibrium path. Note that since $\gamma = \kappa/(\kappa+2) < 1$, condition (27) can be jointly satisfied with $\rho < \delta$, which ensures that providing incentives for search is better than letting the manager shirk.

[10] The result that all targets trigger production when W is sufficiently low relies partially on the assumptions that all targets, regardless of their quality, generate positive return to the investors and require effort and time to be discovered. The former can be justified if the investors have a common sense of the basic properties of investment opportunities worth taken (e.g., firms with strong growth history and healthy balance sheet), and the latter can be interpreted as "no free lunch" in the financial market. If, instead, the $\theta_{\min}$ target represents a "default" option that is always immediately available, then when W is sufficiently low, investor will intuitively abandon the search by resorting to the default option in lieu of contract termination. If $\theta_{\min} < 0$, or if there is a substantial fixed cost for production, then the optimal investment policy may exclude some low-quality targets even when W is low and termination is imminent. If $\theta_{\min}$ is both always available and low-value, the optimal contract may involve random termination in order to peg W at a sufficiently high level to prevent the manager from exploiting this low-value default option, such as the case studied in Varas (2018).

Altogether, the optimal contract during the search stage can be summarized in the following proposition:

Proposition 3 *Under the optimal contract, the investors' value function $V_1(W)$ solves the HJB equation (19) subject to the IC condition (17) and the boundary condition $V_1(0) = 0$. Under Assumption 1, there exist $\{\overline{W}, \underline{W}\}$ that solve (24) and (25), respectively, such that the optimal investment policy $x(W)$ is given by*

$$x(W) = \begin{cases} \theta_{\min}, & \text{if } W < \underline{W} \\ \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(W)\right)\left(\frac{\lambda\gamma\theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}}, & \text{if } \underline{W} \leq W < \overline{W} \\ \left[\frac{\lambda\gamma^2\theta_{\min}^{\kappa}}{\rho(\kappa-2)}\right]^{\frac{1}{\kappa-2}}, & \text{if } W \geq \overline{W} \end{cases} \tag{28}$$

where $x'(W) > 0$ for all $W \in (\underline{W}, \overline{W})$.

Figure 1 illustrates the value function $V_1(W)$ and the three regions of the optimal investment policy $x(W)$. It also plots the first-best investment policy x^{FB} and illustrates the following result:

Corollary 3 *$x(W) < x^{FB}$ for all W.*

To reduce the likelihood of contract termination the investors reduce the investment threshold. When W is sufficiently low, all targets trigger production regardless of their quality. When W is larger, the concern for termination is somewhat eased but is never eliminated. Hence, the optimal investment threshold in the presence of agency frictions is always below the first best, i.e., firms *overinvest*.

5.3 Robustness With Dynamic Adverse Selection

Our model assumes that production is a one-time decision. Once the investment is made and the manager exerts the production effort e, a single output y is realized, and the contracting relationship ends. This implies a static adverse selection problem and simplifies the derivation of the optimal screening contract. This subsection demonstrates the robustness of our results when the production stage is also dynamic and the manager's private information evolves stochastically.

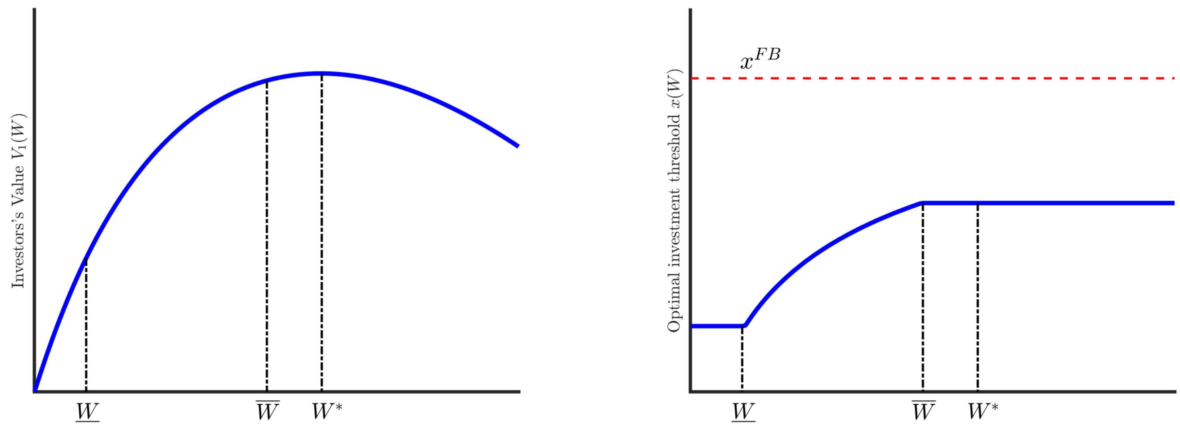

Figure 1: **Investors' Value Function and The Optimal Investment Policy**
The left panel of this figure plots the investors' value function $V_1(W)$ under the optimal contract. The right panel plots the optimal investment threshold $x(W)$ according to Proposition 3 and the first-best investment threshold x^{FB}. $\underline{W}$ and $\overline{W}$ are defined according to (25) and (24), respectively, and $W^* \equiv \arg\max_W V_1(W)$ represents the point at which $V_1(W)$ is maximized. Parameter values are $\lambda = 2.5$, $\kappa = 4.25$, $\delta = 1.1$, $\rho = 0.8$.

Let τ represent the end of the search stage. Consider the following extension: The production stage lasts an exogenous period of $T > 0$ (i.e., from τ to $\tau + T$), during which the manager continuously produces outputs from the target chosen in the previous stage. The production technology is given by

$$y_t = e_t \xi_t \tag{29}$$

where y_t is the output, e_t is the manager's (production) effort exerted at a quadratic personal cost $h(e_t) = e_t^2/2$, and ξ_t is the *productivity* of the target, which now evolves over time. For tractability, we assume the following law of motion for ξ_t:

Assumption 2 *ξ_t follows a geometric Brownian motion (GBM) $d\xi_t = \xi_t (\mu dt + \sigma dZ_t)$ with publicly-known parameters μ and σ. θ determines the initial value of ξ_t, i.e., $\theta = \xi_\tau$.*

The main advantage of this assumption is that, when ξ_t follows a GBM, $\xi_t = \xi_\tau \nu_t$, where

$$\nu_t = \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma Z_t\right], \quad (\nu_\tau = 1) \tag{30}$$

is an exogenous stochastic process with known distribution for any given t. Therefore,

$d\xi_t/d\theta = \xi_t/\xi_\tau = \nu_t$. In other words, the marginal value of target quality θ on its subsequent productivity at any time during the production stage depends on the path of exogenous shocks only, a property that greatly simplifies the analysis below.

If all information is public, the first-best effort and output in the production stage solve

$$\max_{e_t} \; y_t - h(e_t) = e_t \xi_t - h(e_t) \tag{31}$$

The solution is

$$e_t^{FB} = \xi_t; \tag{32}$$

$$y_t^{FB} = \xi_t^2. \tag{33}$$

The investors' expected payoff given the quality of the target, denoted by $V_2^{FB}(\theta)$, is given by

$$V_2^{FB}(\theta) = \mathrm{E} \int_\tau^{\tau+T} \left(y_t^{FB} - \frac{1}{2} \left(e_t^{FB} \right)^2 \right) dt = \frac{\phi}{2} \theta^2, \tag{34}$$

where

$$\phi = \mathrm{E} \int_\tau^{\tau+T} \nu_t^2 dt = \frac{e^{(2\mu+\sigma^2)T} - 1}{2\mu + \sigma^2} \tag{35}$$

is a constant. Here, ϕ thus measures the marginal value of target quality summarizing the joint effect of μ, σ, and T, and will be treated as a known parameter in the subsequent analysis. The first-best investment policy is still a cutoff quality

$$x^{FB} = \left[\frac{\lambda \phi \theta_{\min}^\kappa}{\delta(\kappa - 2)} \right]^{\frac{1}{\kappa-2}}. \tag{36}$$

Investors never abandon the search, and production begins only when a target that clears x^{FB} arrives. Their maximal expect payoff at the beginning of the search stage is

$$V_1^{FB}(x^{FB}) = \frac{\kappa \left(x^{FB} \right)^2}{2(\kappa - 2)} - \frac{\delta \left(x^{FB} \right)^\kappa}{\lambda \theta_{\min}^\kappa}. \tag{37}$$

Similar to the main model, an adverse selection problem arises if effort and productivity are both the manager's private information (while the output y_t is still observable to the investors). However, this adverse selection is now dynamic in nature, because the investors must solicit the truthful reporting of ξ_t for the entire duration of the production stage. Let $\widehat{\theta}$ and $\{\widehat{\xi}_t\}_{t\in(\tau,\tau+T]}$ represent the manager's *reported* target quality and time-t productivity, respectively. The resulting screening contract now involves a series of the output target $\{y_t^{\widehat{\theta}}(\widehat{\xi}_t)\}_{t\in(\tau,\tau+T]}$ and the corresponding wage $\{w_t^{\widehat{\theta}}(\widehat{\xi}_t)\}_{t\in(\tau,\tau+T]}$ if the required output is produced. Conditional on any utility W_{τ^-} carried over from the search stage, the manager's objective is to maximize her expected wage minus her (production) effort cost from the contract – her *information rent* – which is given by

$$R(\theta) = \max_{\widehat{\theta},\widehat{\xi}_t,e_t} \mathrm{E}\left[\int_\tau^{\tau+T} (w_t^{\widehat{\theta}}(\widehat{\xi}_t) - h(e_t))dt\right] \tag{38}$$

subject to the constraint that $e_t\xi_t = y_t^{\widehat{\theta}}(\widehat{\xi}_t)$. That is, similar to that in the baseline model, if the manager reports $\widehat{\xi}_t$, she must produce the required output by exerting a certain degree of effort. The investors' objective is to maximize the expected output net of the manager's wage from the contract. Following a similar argument as that used in the main model, Θ, the set of targets that trigger the investment, is again an open set bounded from below, i.e.,

$$\Theta = \{\theta : \theta \geq x\} \tag{39}$$

for some cutoff threshold x. Then, the investors' objective is to maximize their payoff at the outset of the production stage, which is $V_2(x) - W_{\tau^-}$, where

$$V_2(x) = \max_{y_t,w_t} \int_x^{+\infty} \mathrm{E}\left[\int_\tau^{\tau+T} (y_t - w_t)dt\right] dF(\theta) \tag{40}$$

subject to the IC constraint $\widehat{\theta} = \theta$ and $\widehat{\xi}_t = \xi_t$ for all $t \in (\tau, \tau+T]$.

Comparing to the existing literature, a theoretical innovation (and challenge) of this setting is that the manager's private information ξ_t is *persistent*, which implies the adverse selection problem the investors face is *dynamic*. While there are studies exploring persistent private information in the context of dynamic moral hazard (Williams, 2011, 2015, He et al., 2017, Marinovic and Varas, 2019, and Feng, 2022), studies of persistent private information in the context of adverse selection are rare, as it is known to be a challenging problem.

Fortunately, the specific structures of our model implies that the screening problem during the production stage is also *time-separable*. That is, the set of feasible contract terms (i.e., $\{y_t, w_t\}$) at time $t \in [\tau, \tau + T]$ is independent of the history of the contract, and the flow utility of the manager and the investors at time t depends only on the initial and the current private information of the manager. Under time-separability, the dynamic adverse selection problem can be converted into a static mechanism design problem similar to that analyzed in Section 5.1, allowing us to uniquely pin down $R(\theta)$ and $V_2(x)$ under any incentive compatible contract which is all we need to feed back into the search stage. The resulting optimal contract under this extension is summarized as follows:

Proposition 4 *Under Assumption 2, for any given investment policy x, the optimal contract in the production stage has the following properties:*

- *The manager's information rent from a target of quality $\theta > x$ at the beginning of the production stage is given by*

$$R(\theta) = \frac{\phi\gamma^2}{2}(\theta^2 - x^2). \tag{41}$$

- *The investor's expected payoff at the beginning of the production stage is*

$$V_2(x) = \frac{\phi\gamma\kappa}{2(\kappa - 2)}x^2. \tag{42}$$

- *During the production stage, the investors' optimal output target $\{y_t^*\}_{t\in[0,T]}$ is given by $y_t^* = \gamma\xi_t^2$ and the implied equilibrium production effort is $e_t^* = \gamma\xi_t$.*

The optimal contract during the search stage can be summarized by the investors' value function $V_1(W)$, which solves an HJB equation analogous to (19) subject to the boundary condition $V_2(0) = 0$. In particular, if Assumption 1 holds, and $\phi \geq 1$, then there exist

$\{\overline{W}, \underline{W}\}$ *such that the optimal investment policy* $x(W)$ *is given by*

$$x(W) = \begin{cases} \theta_{\min}, & \textit{if } W < \underline{W} \\ \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(W)\right)\left(\frac{\phi\lambda\gamma\theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}}, & \textit{if } \underline{W} \leq W < \overline{W} \\ \left[\frac{\phi\lambda\gamma^2\theta_{\min}^{\kappa}}{\rho(\kappa-2)}\right]^{\frac{1}{\kappa-2}}, & \textit{if } W \geq \overline{W} \end{cases} \tag{43}$$

$x(W) < x^{FB}$*, and* $x'(W) > 0$ *for all* $W \in (\underline{W}, \overline{W})$*.*

Despite the dynamic nature of the adverse selection problem, our main results remain qualitatively intact. In particular, the manager's information rent in the production stage is still a quadratic function of the target quality θ and the investor's expected payoff is still a quadratic function of the investment threshold x. In the search stage, W drifts downward in the absence of a suitable target, and the optimal investment threshold x is progressively lower, which leads to overinvestment.

The results in this subsection demonstrate the robustness of the main model and its practical implications. Nevertheless, the solution technique of this dynamic version of the adverse selection problem is far more involved than the one used in the static version and is potentially applicable to a broad class of questions involving persistent and time-varying private information. Our solution method utilizes the Myersonian approach developed in Eső and Szentes (2007) and Pavan, Segal, and Toikka (2014) but extended to continuous time. The details are provided in the proof of Proposition 4 in the appendix for the interested readers. We hope that the solution to this extension provides a unified framework for researchers interested in jointly studying these two important agency frictions.

6 The Effect of Interaction Between Agency Frictions

The core problem analyzed in this paper is the interaction of two agency frictions: moral hazard and adverse selection. To further illustrate the effect of such interaction, it is useful to compare the results of this model with those in models with only one of the frictions.

Consider a benchmark setting with a single type of target available for investment. The gross return of the target to the investors is K (a constant), which is realized immediately when the target is found and revealed to the investors. The search cost (δ) and the arrival intensity of the target if the manager works (λ) are the same as before. This represents only a problem of moral hazard (regarding the unobservable search effort) and the following proposition summarizes the optimal contract under this benchmark setting:

Proposition 5 *Suppose there is a single target worth K arriving via a Poisson process with intensity λ when the manager exerts search effort. Then, under the optimal contract,*

$$dW_t = \frac{\rho}{\lambda}(dN_t - \lambda dt) \tag{44}$$

Define $V_2 = K - \rho/\lambda$. The investors' value function $V_1(W)$ solves the HJB equation

$$0 = -\delta - \rho V_1'(W) + \lambda \left[V_2 - W - V_1(W)\right] \tag{45}$$

plus the boundary conditions $V_1(0) = 0$.

The optimal contract described in Proposition 5 has different properties than those described in Proposition 3. Most crucially, without the distribution of target quality, investors do not have the choice of investment policy, and the search ends as soon as the target arrives. The resulting IC constraint for search effort is always binding along the equilibrium path. This is because the rate at which W drifts down is only subject to the IC constraint and a higher rate of the drift carries two costs. First, it expedites termination in the absence of the target. Second, it must be compensated with a larger upward jump in W when the target arrives, which lowers the investors' payoff.

A benchmark setting only with adverse selection can be found in Malenko (2019). This paper studies the optimal design of a dynamic capital allocation process in which a division manager privately observes the arrival and quality of investment projects. There is no moral hazard because project arrivals are stochastic and do not depend on the manager's effort. The optimal contract focuses on soliciting truthful report of the manager's private information, which is transient. Each project is a take-it-or-leave-it opportunity with instant return if the headquarters undertake it, and there is an auditing technology that (for the most part) perfectly reveals the quality of the project at a cost. The agency friction arises from the manager's empire-building preference: she is inclined to exaggerate the quality of the project in order to induce a larger investment. As a result, her continuation utility drifts *upward* in the absence of any reported project and jumps *downward* when an investment project is taken without auditing to cancel out her private benefit from the investment. There is no termination under the optimal contract, because the headquarters can always prevent W from dropping too low by shrinking the size of the investment. Finally, the optimal contract always induces underinvestment.

In contrast, investors in our model face a moral hazard problem of unobservable search effort in addition to unobservable target arrival and quality. As a result, the manager's continuation utility drifts *downward* in the absence of a suitable target and jumps *upward* when investment is made.[11] In particular, because the manager can divert the search resources to generate private benefits, the search stage must end at some point, otherwise the manager will never report the arrival of any target and enjoy unlimited utility. Moreover, with her private information regarding the target quality, the manager can continue to extract rents from the investors during the production stage. Finally, the optimal contract always induces overinvestment and yields different empirical implications than Malenko (2019).

7 Comparative Statics and Empirical Predictions

The simple form of the optimal investment policies x^{FB} and $x(W)$ summarized in Lemma 1 and Proposition 3 allows the derivation of useful comparative statics. According to Corollary 3, agency frictions in the model lead to overinvestment, the degree of which can be measured in at least two ways: (i) the ratio between $x(\underline{W})$ and x^{FB} (i.e., $\theta_{\min}/x^{FB}$) which represents the *maximal* degree of overinvestment; and (ii) the ratio between $x(\overline{W})$ and x^{FB} (i.e., $\bar{x}/x^{FB}$) which represents the *minimal* degree of overinvestment.[12] A parameter change is said to exacerbate overinvestment if it lowers at least one of the two ratios. Based on these

[11] In that regard this result resembles Che, Iossa, and Rey (2021) where an uninformed principal uses another follow-on contract to induce the truthful report from an informed agent about the cost of implementing an innovation idea. However, both the generation and the implementation of the idea in Che, Iossa, and Rey (2021) are static problems. In contrast, the search stage in this paper involves a dynamic problem with endogenous termination.

[12] Note that condition (27) ensures $x(W^*) = \bar{x}$. Thus, the minimal degree of overinvestment is also equivalent to the initial level of overinvestment at the outset of the search stage.

definitions, Lemma 1 and Proposition 3 imply the following:

Proposition 6 *A higher λ or ρ and a lower δ exacerbate overinvestment.*

Intuitively, as discussed in Section 4, a higher target arrival rate λ and a lower search cost δ increases x^{FB} and reduces the ratio $\theta_{\min}/x^{FB}$, thereby exacerbating the maximal degree of overinvestment. The search cost δ does not affect $\bar{x}$ and thus a lower δ also decreases $\bar{x}/x^{FB}$ and exacerbates the minimal degree of overinvestment. Meanwhile, the tightness of the IC condition depends on the manager's private benefit from shirking ρ. A higher ρ tightens the IC condition and lower $\bar{x}$ but does not affect x^{FB}, thus exacerbating the minimal degree of overinvestment.

These comparative statics generate empirically relevant predictions that are testable in the markets of M&A, HFA, VC or SPAC, in which there is extensive empirical evidence for overinvestment. Prior studies (e.g., Shleifer and Vishny, 1997; Franzoni, 2009; Malenko, 2019; Gregor and Michaeli, 2020, 2022) often attribute overinvestment to the manager's empire-building preferences. Our model offers an alternative explanation based on optimal contracting under agency frictions. In particular, Proposition 6 implies that overinvestment is positively correlated with λ and ρ and negatively correlated with δ. Empirically, the intensity λ of target arrival, can be proxied by the number of firms in the industry and/or the frequency of M&A, HFA, VC, or SPAC activities. The manager's private benefit/perk ρ is harder to measure directly. However, in our model it equals the minimal speed at which the manager's continuation utility has to drift down without the target. Therefore, ρ can be indirectly measured by the incentive power of the managerial contract such as the fraction of inside equity. This is also the standard interpretation in the optimal contracting literature (e.g., DeMarzo and Sannikov, 2006). The search cost δ can be proxied by geographical proximity/location (of the bidder, activist hedge fund, VC fund, or SPAC), executive connections, as well as by standard measures for availability of information about investment targets (e.g., percentage of public firms in the industry, stock liquidity, analyst coverage, and institutional holdings) given that more information facilitates searching. Together, the results in Proposition 6 can be translated into the following testable empirical prediction:

Prediction 1 *Overinvestment in M&A, HFA, VC or SPAC is positively associated with the* *number of firms and frequency of deals in these markets, the geographical proximity and the average incentive power (of the bidder, managers of activist hedge fund, VC fund, or SPAC), as well as with the executive connections, stock liquidity, analyst coverage, and institutional holdings of the targets.*

Our model also predicts that along the equilibrium path, investors optimally adopt a progressively lower investment threshold with value between $x(\overline{W})$ and $x(\underline{W})$. Because a higher investment threshold is associated with higher expected payoff ($V_2'(x) > 0$), the dynamics of x can be potentially proxied by the variations in the returns to M&A deals, VC investments, HFA targets, or SPAC business combinations. In particular, the gap between $x(\overline{W})$ and $x(\underline{W})$ can be interpreted as the return *dispersion* in those markets, which is arguably straightforward to measure. Thus, Propositions 3 and 6 suggest that such dispersion is wider if λ is higher or if ρ is lower. Therefore, Proposition 6 implies the following testable empirical prediction:

Prediction 2 *The return dispersion of M&A deals, VC investments, HFA targets, or SPAC business combinations is positively associated with the frequency of those activities and the number of firms in the market. The return dispersion is negatively associated with the average incentive power of the managerial contract of the bidder, activist hedge fund, VC fund, or SPAC.*

We further complement the analytical comparative statics with numerical simulations, which yield practical predictions regarding the *distribution* of the variables of interest. Specifically, for each set of parameters, we simulate 1,000 paths of evolutions of the contract and calculate the success rate (i.e., the fraction of paths in which a suitable target arrives and triggers the investment) and the manager's initial time budget (i.e., the maximum search time allowed before termination). We also calculate the average and standard deviations of the search time, target value, and managerial compensation conditional on the investment being triggered.

Table 1 presents the results for a benchmark case and several comparative statics in which we maintain the value for all but one parameter of the benchmark. A higher λ (target arrival rate) increases both the frequency of deal completion and the maximal search time allowed before termination. In contrast, a higher δ (search cost) or ρ (agency cost) both lower the completion rate and the maximal time allowed. A higher κ, which means a thinner tail for the underlying distribution of target quality (i.e., fewer high-quality targets), imply the same results.

Table 1: **Simulation**

	(1) Benchmark	(2) Higher λ	(3) Higher ρ	(4) Higher δ	(5) Higher κ
Success rate	0.33	0.44	0.28	0.29	0.26
Initial time budget	143	446	60	105	61
Conditional on search being successful					
Average search time	78.04	255.97	30.81	62.42	33.40
Std. Dev.	43.65	144.23	17.97	30.96	18.53
Average target value	5.47	11.02	4.60	5.41	3.94
Std. Dev.	38.82	44.43	41.84	41.11	31.54
Average managerial compensation	3.33	6.63	2.88	3.21	2.47
Std. Dev.	24.28	27.87	26.14	25.69	20.50

The parameters for the benchmark are $\lambda = 3$, $\rho = 0.8$, $\delta = 1.1$, $\kappa = 2.5$. In columns (2) to (5), all parameters are the same as those in the benchmark except for: $\lambda = 4$ in column (2); $\rho = 1$ column (3); $\delta = 1.3$ in the column (4); and $\kappa = 2.6$ in column (5). Each column corresponds to 1,000 paths of simulations. Success rate is the fraction of the paths in which a suitable target according to the optimal investment policy arrives and investment is triggered. The initial time budget is the maximal search time allowed before contract termination. Managerial compensation of each deal refers to $W_{\tau^-} + R(\theta)$, i.e., the residual utility carried over from the search stage plus the managerial rent in the production stage based on the quality of the target. The average and standard deviations of the search time, target value, and managerial compensation are conditional moments of all paths that do not end with termination.

Interestingly, higher λ implies an average *longer* search time conditional on the search being successful. There are two reasons for this outcome. First, higher λ increases the maximal allowed search time. Second, with more abundant potential targets, the optimal contract imposes a higher initial hurdle for investment that also declines slowly over time. This can be seen in the higher average target value and managerial compensation, which are determined by the investment hurdle x. In comparison, the manager is given a shorter time budget as well as a more rapidly declining investment hurdle when δ, ρ, or κ are higher, resulting in a lower average target value and managerial compensation but also a faster search time. These results can be summarized in the following testable prediction, which may help reconcile the observed correlation between the success rate and performance of various search processes, such as the large numbers of SPAC business combinations completed in recent years and their poor subsequent returns.[13]

Prediction 3 *The average and the variance of the returns to M&A deals, HFA targets, VC investments and SPAC business combinations are positively correlated with geographical proximity, frequency of deal completion, number of firms and relative frequency of public firms, executive connections, stock liquidity, analyst coverage, and institutional holdings of the targets, and negatively correlated with the average incentive power of the managerial contract of the bidder, activist hedge fund, VC fund, or SPAC.*

The empirical implications of our extension with dynamic adverse selection are similar to those in the baseline model with the addition of a new one pertaining to the parameter ϕ, which can be interpreted as the industry or regional average return of M&A, HFA, VC, or SPAC activities. It is straightforward to see that a higher ϕ increases both x^{FB} and $x(\overline{W})$ while $x(\underline{W}) = \theta_{\min}$ is unchanged, leading to the following testable prediction:

Prediction 4 *The overinvestment and return dispersion in M&A, HFA, VC, or SPAC are positively associated with the average returns in those markets.*

It is worth emphasizing that the above-described hypotheses are formulated *ceteris paribus.* Empirical testing of these hypotheses thus requires identification to control for confounding factors. Although rigorous empirical analysis is outside the scope of this paper, several identification strategies, such as using the decimalization on major stock exchanges as an exogenous shock to stock liquidity (e.g., Edmans, Fang, and Zur, 2013), or the addition of non-stop flights between the locations of a firm and its potential investment targets as an exogenous shock to search cost (e.g., Bernstein, Giroud, and Townsend, 2016), already exist in the literature and may provide useful settings to explore the predictive power of the model in this paper.

[13]See, e.g., Gahng et al. (2022) and Klausner, Ohlrogge, and Ruan (2022) for relevant statistics.

8 Concluding Remarks

We consider a setting where investors delegate the search for investment targets and their operation to a manager. Our model highlights two novel yet realistic features of the search process. First, the manager is privy to information about target arrival and quality and needs to receive proper incentives to maximize the likelihood of discovering investment targets and truthfully disclose their value to investors. Second, the relationship between the investors and the manager does not end when the relevant information is disclosed. The manager is also tasked with generating output from the investment target—a process in which the manager's information advantage pertains. These features lead to simultaneous presence of an adverse selection and a moral hazard problem. The resulting optimal investment policies exhibit overinvestment relative to the first-best, which sheds light on understanding and predicting the returns from M&A deals, VC investments, HFA interventions, and/or SPAC business combinations.

Our work can be extended in several directions. For simplicity, the model does not allow the manager to revert to the search stage once production begins. In practice, the search and production processes do not always move forward linearly. Letting the manager conduct both search and production repeatedly or simultaneously may yield interesting insights about firms' optimal internal organization and/or resource allocation. The manager may be allowed to exert variable levels of effort in order to expedite the search process. Finally, the investors may also have access to an auditing technology which can reveal the quality of the announced target at a cost. We leave these topics for future work.

Appendix

Proof of Lemma 1: First, $V_2^{FB\prime}(\theta) = \theta > 0$. Therefore, if $x \in \Theta^{FB}$ for some x, then $\theta \in \Theta^{FB}$ for all $\theta > x$. i.e., $\Theta^{FB} = \{\theta : \theta \geq x^{FB}\}$. Thus, the conditional expectation of the investors' payoff from setting an arbitrary cutoff investment quality x is

$$\mathcal{V}_2^{FB}(x) \equiv \mathrm{E}\left[V_2^{FB}(\theta)|\theta \geq x\right] = \int_x^{+\infty} \frac{\theta^2}{2}\left(\frac{\kappa x^\kappa}{\theta^{\kappa+1}}\right) d\theta = \frac{\kappa x^2}{2(\kappa-2)}. \tag{46}$$

Equation (46) utilizes the fact that a Pareto distribution truncated from below at some $x > \theta_{\min}$ is a Pareto distribution with the same shape parameter κ and scale parameter x. Equation (46) also reveals why $\kappa > 2$ is needed for the first-best to exist. Let $V_1^{FB}(x)$ be the investors' value function at the outset of the search stage associated with cutoff policy x, then

$$V_1^{FB}(x) = \max_x \int_0^{+\infty} \left[-\delta t + F(x)V_1^{FB}(x) + (1-F(x))\mathcal{V}_2^{FB}(x)\right] \lambda e^{-\lambda t} dt. \tag{47}$$

The three terms inside the square brackets represent the cost of search, the payoff from a target with quality lower than x, and the expected payoff from the arrival of a target with quality x or above, respectively. Using the fact that $F(x) = 1 - (\theta_{\min}/x)^\kappa$ for Pareto distribution, $V_1^{FB}(x)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$0 = \max_x \; -\delta + \lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa \left[\mathcal{V}_2^{FB}(x) - V_1^{FB}(x)\right]. \tag{48}$$

Substituting (46) into the HJB equation and re-arranging the terms yields

$$V_1^{FB}(x) = \max_x \; \frac{\kappa x^2}{2(\kappa-2)} - \frac{\delta x^\kappa}{\lambda \theta_{\min}^k}. \tag{49}$$

The first order condition with respect to x yields x^{FB} as in (4). Substituting x^{FB} into (49) yields $V_1^{FB}(x^{FB})$ as in (5).

Proof of Lemma 2: Suppose there is an incentive-compatible optimal contract $\mathcal{C}$ under which an open set H exists with the following properties: for all $\theta \in H$, $\theta \notin \Theta$, and there exists θ' such that $\theta' \in \Theta$ but $\theta' < \tilde{\theta} \equiv \inf H$. Let $\bar{\theta} \equiv \max\{\theta : \theta \in \Theta, \theta < \tilde{\theta}\}$. Clearly, $y(\bar{\theta}) - w(\bar{\theta}) > 0$ and $w(\bar{\theta}) \geq h(e(\bar{\theta}))$ if $\mathcal{C}$ is optimal. Now consider a contract $\mathcal{C}'$ that is otherwise identical to $\mathcal{C}$ except for the following: for any report $\widehat{\theta} \in H$, the required output $y(\widehat{\theta}) = y(\bar{\theta})$ and the associated compensation is $w(\widehat{\theta}) = w(\bar{\theta})$. This contract is incentive-compatible because for all $\tilde{\theta} \in H$, $e(\tilde{\theta}) = y(\bar{\theta})/\tilde{\theta} < e(\bar{\theta})$ and is independent of the report $\widehat{\theta}$. Thus, $w(\tilde{\theta}) = w(\bar{\theta}) > h(e(\tilde{\theta}))$ for all $\tilde{\theta}$. However, $y(\bar{\theta}) - w(\bar{\theta}) > 0$ implies that $\mathcal{C}'$ generates the same payoff as $\mathcal{C}$ for all $\theta \notin H$ but positive (higher) payoff for all $\tilde{\theta} \in H$, contradicting the assumption that $\mathcal{C}$ is optimal. Therefore, it must be that such H does not exist under the optimal contract.

Proof of Proposition 1: The investors offer a screening contract $\{w(\widehat{\theta}), y(\widehat{\theta})\}$. The manager reports her type $\widehat{\theta}$, produces the required level of output, and receives the associated compensation. With a slight abuse of notation, define

$$R(\theta, \widehat{\theta}) = w(\widehat{\theta}) - h(e) \tag{50}$$

as the information rent of the manager with type-θ reporting $\widehat{\theta}$, subject to the constraint that $y(\theta, e) = y(\widehat{\theta})$. i.e., she must produce the level of output designed for the type-$\widehat{\theta}$ agent. Let $e(y, \theta)$ represent the necessary effort required by a type-θ manager to produce output y. Then, one can define $R(\theta) = R(\theta, \theta)$ as the agent's equilibrium rent under truthful reporting, and

$$\widehat{\theta}^*(\theta) = \arg\max_{\widehat{\theta}} \; R(\theta, \widehat{\theta}) \tag{51}$$

as the optimal reported type chosen by a type-θ agent. This optimality implies the following *envelope condition*

$$R_{\widehat{\theta}}(\theta, \widehat{\theta}^*(\theta)) = 0. \tag{52}$$

Therefore, in the equilibrium

$$R'(\theta) = \frac{\partial R(\theta, \widehat{\theta}^*(\theta))}{\partial \theta} = R_\theta + R_{\widehat{\theta}}(\theta, \widehat{\theta}^*(\theta)) \frac{d\widehat{\theta}^*(\theta)}{d\theta} = R_\theta = -h'(e) e_\theta(y, \theta) \tag{53}$$

based on the envelope condition.

The investors' payoff in the production stage is therefore $V_2(x) - W_{t^-}$, where

$$V_2(x) = \max_{y,w} \int_x^\infty [y(\theta) - w(\theta)] dF(\theta) = \max_{y,w} \int_x^\infty [y(\theta) - h(e(\theta, y)) - R(\theta)] dF(\theta) \tag{54}$$

where

$$F(\theta) = 1 - \left(\frac{x}{\theta}\right)^\kappa \tag{55}$$

$$f(\theta) = \frac{\kappa x^\kappa}{\theta^{\kappa+1}}. \tag{56}$$

Applying integration by parts to the last term inside the integral of $V_2(x)$ in (54) yields

$$\int_x^\infty R(\theta) dF(\theta) = \int_x^\infty R'(\theta)(1 - F(\theta)) d\theta + R(x). \tag{57}$$

Substituting this into (54) above yields

$$V_2(x) = \max_y \int_x^\infty [y - h(e) - R'(\theta) g(\theta)] f(\theta) d\theta \tag{58}$$

where

$$g(\theta) \equiv \frac{1 - F(\theta)}{f(\theta)}$$

represents the inverse hazard function of θ. Replacing $R'(\theta)$ with (53), point-wise maximization with respect to y yields the following optimality condition:

$$1 - h'(e)e_y(e, \theta) + \frac{dh'(e)e_\theta(y, \theta)}{dy} g(\theta) = 0 \tag{59}$$

which yields the optimal target y for each type θ. Because $y = \theta e$ and $h(e) = e^2/2$, $e(y, \theta) = y/\theta$, $e_\theta = -y/\theta^2$, and

$$\frac{dh'(e)e_\theta(y, \theta)}{dy} = -\frac{dy^2/\theta^3}{dy} = -\frac{2y}{\theta^3}.$$

The fact that θ follows a Pareto distribution implies that

$$g(\theta) = \frac{1 - F(\theta)}{f(\theta)} = \frac{\theta}{\kappa} \tag{60}$$

Substituting these results into (59) yields

$$1 - \frac{y}{\theta^2} - \frac{2y}{\kappa\theta^2} = 0 \tag{61}$$

which implies $y = \gamma\theta^2$, where

$$\gamma = \frac{\kappa}{\kappa + 2}. \tag{62}$$

Substituting $y = \gamma\theta^2$ into (50). The IC constraint $\widehat{\theta} = \theta$ and the fact that $R(x) = 0$ under the optimal screening contract yields

$$R(\theta) = \frac{\gamma^2}{2}(\theta^2 - x^2). \tag{63}$$

Combine this with $y = \gamma\theta^2$ implies that

$$V_2(x) = \frac{\gamma\kappa}{2(\kappa - 2)} x^2. \tag{64}$$

Proof of Proposition 2: Let $\mathcal{F}_t$ denote the filtration generated by the manager's report θ_t (where $\theta_t = 0$ if no investment opportunity arrives). W_t is an $\mathcal{F}_t$-martingale and thus, by the martingale representation theorem for jump processes, there exists a $\mathcal{F}_t$-predictable, integrable process β_t such that

$$dW_t = a_t\beta_t(dN_t - \lambda(1 - F(x_t))dt). \tag{65}$$

Incentive compatibility of the search effort requires that $\lambda(1-F(x_t))\beta_t \geq \rho$. Incentive compatibility of truthful reporting of θ requires that $W_\tau - W_{\tau^-} = R(\theta_\tau)$ if the contract moves to the next stage, which implies $\beta_t = \mathrm{E}[R(\theta_t)|\theta_t \geq x_t] = U(x_t)$ by the property of a martingale.[14] Thus, given the investment policy x_t,

$$dW_t = U(x_t)(dN_t - \lambda(1-F(x_t))dt) \tag{66}$$

where $U(x_t)\lambda(1-F(x_t)) \geq \rho$. Substituting in $1-F(x_t) = \left(\frac{\theta_{\min}}{x_t}\right)^\kappa$ yields equations (16) and (17).

Proof of Proposition 3: Applying Ito's lemma to dW_t implies the investors' value function in the search stage solves the following HJB equation:

$$0 = \max_x \quad -\delta - \lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa U(x)V_1'(W) + \lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa [V_2(x) - W - V_1(W)] \tag{67}$$

subject to the IC constraint (17). Substituting $U(x)$ from (13) and $V_2(x)$ from (12) into the HJB equation and rearranging terms yields:

$$V_1(W) = \max_x \quad \left(\frac{\gamma\kappa}{2(\kappa-2)}\right)x^2 - \left(\frac{\gamma^2 x^2}{\kappa-2}\right)V_1'(W) - W - \frac{\delta}{\lambda}x^\kappa\theta_{\min}^{-\kappa}. \tag{68}$$

Suppose the IC constraint is slack, then the first order condition implies

$$\frac{\gamma}{\kappa-2}\left[\kappa - 2\gamma V_1'(W)\right]x = \frac{\delta}{\lambda}\kappa x^{\kappa-1}\theta_{\min}^{-\kappa}. \tag{69}$$

The solution is

$$x(W) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right)V_1'(W)\right)\left(\frac{\lambda\gamma\theta_{\min}^\kappa}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}}. \tag{70}$$

If (17) is binding, then

$$\lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa U(x) = \frac{\lambda\theta_{\min}^\kappa}{x^\kappa}\left(\frac{\gamma^2 x^2}{\kappa-2}\right) = \rho \tag{71}$$

yields the solution

$$x(W) = \bar{x} \equiv \left[\frac{\lambda\gamma^2\theta_{\min}^\kappa}{\rho(\kappa-2)}\right]^{\frac{1}{\kappa-2}}. \tag{72}$$

[14] Note that this also follows the definition of $R(\theta)$ in Section 5.1 which is the information rent the manager must be given to reveal θ truthfully *in addition to* any utility W_{τ^-} carried over from the search stage.

Substituting (72) into (70) implies that there exists $\overline{W}$ such that

$$x(\overline{W}) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(\overline{W})\right)\left(\frac{\lambda\gamma\theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}} = \bar{x}. \tag{73}$$

That is, $\overline{W}$ solves

$$V_1'(\overline{W}) = \frac{\kappa}{2}\left(\frac{1}{\gamma} - \frac{\delta}{\rho}\right). \tag{74}$$

Meanwhile, substituting $x = \theta_{\min}$ into (70) implies that there exists $\underline{W}$ such that

$$\theta_{\min} = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(\underline{W})\right)\left(\frac{\lambda\gamma\theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}}. \tag{75}$$

That is, $\underline{W}$ solves

$$V_1'(\underline{W}) = \left(1 - \frac{\delta(\kappa-2)}{\gamma\lambda\theta_{\min}^2}\right)\frac{\kappa}{2\gamma}. \tag{76}$$

The existence of both $\{\underline{W}, \overline{W}\}$ requires that $\overline{W} > \underline{W}$, which is equivalent to

$$\frac{\kappa}{2}\left(\frac{1}{\gamma} - \frac{\delta}{\rho}\right) < \left(1 - \frac{\delta(\kappa-2)}{\gamma\lambda\theta_{\min}^2}\right)\frac{\kappa}{2\gamma} \tag{77}$$

which simplifies to (26). Finally, $V_1'(\overline{W}) > 0$ requires that

$$\frac{\kappa}{2}\left(\frac{1}{\gamma} - \frac{\delta}{\rho}\right) > 0 \tag{78}$$

which implies (27).

Proof of Corollary 3: We can prove this result regardless of whether Assumption 1 holds, i.e., whether the IC constraint is binding for some $\overline{W}$. Note that under the optimal contract, $V'(W) > -1$ for all W. This is because investors can always make a cash transfer to the manager, which lowers W and $V(W)$ by the exact same amount. Therefore, the marginal value of building W inside the firm can never be lower than the marginal value of cash transfer, which is -1. Substituting $V'(W) = -1$ into the first-order condition (22) implies that

$$\lim_{W\to+\infty} x(W) = \left[\gamma\left(1 + \frac{2\gamma}{\kappa}\right)\left(\frac{\lambda\theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}} \tag{79}$$

if the IC constraint (17) is never binding. Because

$$\frac{2\gamma}{\kappa} = \frac{2\kappa}{\kappa(\kappa+2)} < 1 \tag{80}$$

when $\kappa > 2$, this limit is always smaller than x^{FB}. Thus, $x(W) < x^{FB}$ for all W.

Proof of Proposition 4:

A. The Production Stage

This section solves the optimal screening contract in the production stage. The proof begins with deriving the manager's information rent for *any* incentive compatible contract. Consider any report $\widehat{\theta}$ made by a manager possessing an arbitrary θ-quality target. Based on this report, the manager is assigned the contract $\mathcal{C}(\widehat{\theta})$, which imposes output target $y^{\widehat{\theta}}(\widehat{\xi}_t)_t$ for any future report $\widehat{\xi}_t$, the associated wage $w^{\widehat{\theta}}(\widehat{\xi}_t)_t$ if the required output is produced. The contract implies a recommended effort process $\widehat{e}_t \equiv y_t^{\widehat{\theta}}/\widehat{\xi}_t$ for all t. Therefore, given the true productivity process ξ_t, the manager's actual effort choice e_t must satisfy:

$$e_t \xi_t = \widehat{e}_t \widehat{\xi}_t. \tag{81}$$

The payoff for the manager is

$$R(\theta; \widehat{\theta}) = \mathrm{E}\left[\int_{\tau}^{\tau+T} \left(w^{\widehat{\theta}}(\widehat{\xi}_t) - h(e_t)\right) dt\right]. \tag{82}$$

In principle, $\widehat{\theta}$ represents a very large set of possible deviations of the manager. However, Pavan et al. (2014) and the subsequent studies of dynamic adverse selection (e.g., Bergemann and Strack, 2015, Gershkov, Moldovanu, and Strack, 2018, etc.) show that if the screening problem is time-separable, it is without the loss of generality to establish the IC condition for a particular type of deviation from the manager: if she misreports the target quality, $\widehat{\theta} \neq \theta$, her follow-up strategy is to continue misreporting *as if the true quality was* $\widehat{\theta}$ *and she had reported that truthfully.* More formally, at any time, the manager's reported productivity satisfies the following so-called *consistent deviation*:

$$\widehat{\xi}_t = \widehat{\theta} v_t = \widehat{\theta} \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma Z_t\right] \tag{83}$$

where Z_t represents the true productivity shocks the misreporting manager experiences. This implies that although the manager's private information regarding ξ_t is persistent, it is without loss of generality to label each manager only by the quality of her target θ. Thus, we can differentiate (82) with respect to θ to obtain:

$$\frac{\partial}{\partial\theta}R(\theta;\widehat{\theta}) = \mathrm{E}\left[\int_{\tau}^{\tau+T}\left(-\frac{\partial}{\partial\theta}h(e_t)\right)dt\right] \tag{84}$$

$$= \mathrm{E}\left[\int_{\tau}^{\tau+T}\left(\frac{e_t\widehat{e}_t\widehat{\xi}_t}{\theta\xi_t}\right)dt\right] \tag{85}$$

$$= \mathrm{E}\left[\int_{\tau}^{\tau+T}\left(\frac{e_t\widehat{e}_t\widehat{\theta}}{\theta^2}\right)dt\right] \tag{86}$$

where the second line utilizes the constraint (81), and the third line utilizes the consistent deviation (83). Evaluating (86) at the equilibrium ($\widehat{e}_t = e_t, \widehat{\xi}_t = \xi_t$) and substituting e_t with y_t/ξ_t implies

$$R'(\theta) = \mathrm{E}\left[\int_{\tau}^{\tau+T}\left(\frac{e_t^2}{\theta}\right)dt\right] = \mathrm{E}\left[\int_{\tau}^{\tau+T}\frac{1}{\theta}\left(\frac{y_t}{\xi_t}\right)^2 dt\right] \tag{87}$$

which is the *dynamic envelop condition* analogous to the envelop condition derived in the proof of Proposition 1 above. Integrating (87) from x up yields the information rent for any given θ-quality target:

$$R(\theta) = \int_x^{\theta}\mathrm{E}\left[\int_{\tau}^{\tau+T}\frac{1}{q}\left(\frac{y_t}{\xi_t}\right)^2 dt\right]dq + R(x). \tag{88}$$

Next, we derive the investors' expected payoff given the distribution of θ. With a slight abuse of notation, let $\int_x^{\infty}(\cdot)dF(\theta;x)$ denote the expectation of θ taken under the support Θ taking into account how the distribution of $F(\theta)$ shifts with x. The investors' maximal payoff at the outset of the production stage under any incentive compatible contract is $V_2(x) - W_{\tau^-}$, where

$$V_2(x) = \max_{y_t,w_t}\int_x^{\infty}\mathrm{E}\left[\int_{\tau}^{\tau+T}(y_t - w_t)dt\right]dF(\theta;x). \tag{89}$$

The definition of information rent R (Eq. 38) implies that

$$\mathrm{E}\left[\int_{\tau}^{\tau+T}w_t dt\right] = R(\theta) + \mathrm{E}\left[\int_{\tau}^{\tau+T}h(e_t)dt\right]. \tag{90}$$

Substituting this into the definition of $V_2(x)$ (Eq. 89) yields

$$V_2(x) = \max_{y_t}\int_x^{\infty}\mathrm{E}\left[\int_{\tau}^{\tau+T}\left(y_t - \frac{1}{2}\left(\frac{y_t}{\xi_t}\right)^2\right)dt\right]dF(\theta;x) - \int_x^{\infty}R(\theta)dF(\theta;x). \tag{91}$$

Applying integration by parts and the fundamental theorem of calculus to the last term yields

$$\int_x^\infty R(\theta)dF(\theta;x) = \int_x^\infty R'(\theta)\left(\frac{1-F(\theta;x)}{f(\theta;x)}\right)dF(\theta;x) + R(x) \tag{92}$$

$$= \int_x^\infty R'(\theta)\left(\frac{\theta}{\kappa}\right)dF(\theta;x) + R(x) \tag{93}$$

where the second line comes from the property of the Pareto distribution. Clearly, $R(x) = 0$ under the optimal contract. Replacing $R'(\theta)$ with (87) and substituting the above term back to (91) yields

$$V_2(x) = \max_{y_t} \int_x^{+\infty} \mathrm{E}\left[\int_\tau^{\tau+T}\left(y_t - \left(\frac{1}{2}+\frac{1}{\kappa}\right)\left(\frac{y_t}{\xi_t}\right)^2\right)dt\right]dF(\theta;x). \tag{94}$$

Point-wise maximization of (94) with respect to y_t yields the optimal output target y_t^* and effort e_t^*:

$$y_t^* = \gamma\xi_t^2 \tag{95}$$

$$e_t^* = \gamma\xi_t \tag{96}$$

where $\gamma = \kappa/(\kappa+2)$. Substituting (96) and (95) back into (88) yields the following information rent under the optimal contract:

$$R(\theta) = \int_x^\theta \mathrm{E}\left[\int_\tau^{\tau+T}\frac{1}{q}(\gamma\xi)^2\,dt\right]dq \tag{97}$$

$$R(\theta) = \int_x^\theta \gamma^2 q\mathrm{E}\left[\int_\tau^{\tau+T}\nu_t^2 dt\right]dq \tag{98}$$

$$= \int_x^\theta \phi\gamma^2 q dq = \frac{\phi\gamma^2}{2}\left(\theta^2 - x^2\right) \tag{99}$$

Finally, substituting (95) back into (94) yields

$$V_2(x) = \int_x^{+\infty}\mathrm{E}\left[\int_\tau^{\tau+T}\frac{\gamma\xi_t^2}{2}dt\right]dF(\theta;x) \tag{100}$$

$$= \int_x^{+\infty}\frac{\phi\gamma\kappa x^\kappa}{2}\theta^{1-\kappa}d\theta = \frac{\phi\gamma\kappa}{2(\kappa-2)}x^2. \tag{101}$$

Note that, similar to the baseline model, because the investors and the manager share the same discount rate (both 0), and there is no endogenous turnover during the production stage, all wage payments $\{w_t\}$ can be postponed until the end of the production period. Any W_{τ^-} carried over to the production stage can also be paid at the end of the production stage together with all the accrued wage payments.

B. The Search Stage

Let $a_s \in \{0,1\}$ denote the manager's shirking and working actions, respectively. Let τ

and W_τ denote the stopping time of the search stage (either due to progress to the next stage or contract termination) and the associated promised utility to the manager. Because the investors and the manager share the same discount rate (both 0) and $\rho < \delta$, Lemma 3 still applies. That is, $a_t = 1$ for all $t < \tau$, and there is no intermediate payment during the search stage. The manager's continuation utility in this stage thus can be written as:

$$W_t = E\left[\int_t^\tau \rho(1-a_s)ds + W_\tau\right]. \tag{102}$$

By the martingale representation theorem for jump processes, given any investment strategy x of the investors, there exists a $\mathcal{F}_t$-predictable, integrable process β_t such that

$$dW_t = a_t\beta_t(dN_t - \lambda(1-F(x_t))dt). \tag{103}$$

Incentive compatibility of the search effort requires that $\lambda(1-F(x_t))\beta_t \geq \rho$. Incentive compatibility of truthful reporting of θ requires that $K_\tau - W_\tau = R(\theta_\tau)$ if the contract moves to the next stage, which implies $\beta_t = \mathrm{E}[R(\theta_t)|\theta_t \geq x_t] = U(x_t)$ by the property of a martingale, where

$$U(x) \equiv \mathrm{E}\left[R(\theta)|\theta \geq x\right] = \int_x^{+\infty} \frac{\phi\gamma^2}{2}(\theta^2 - x^2)\left(\frac{\kappa x^\kappa}{\theta^{\kappa+1}}\right)d\theta = \frac{\phi\gamma^2 x^2}{\kappa-2}. \tag{104}$$

Therefore, under an incentive compatible contract with investment policy x_t,

$$dW_t = U(x_t)(dN_t - \lambda(1-F(x_t))dt) \tag{105}$$

where $\lambda(1-F(x_t)) \geq \rho$. Then, Ito's lemma implies the investors' value function in the search stage solves the HJB equation:

$$0 = \max_x \quad -\delta - \lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa U(x)V_1'(W) + \lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa [V_2(x) - W - V_1(W)] \tag{106}$$

subject to the IC constraint (17). Substituting $U(x)$ from (104) and $V_2(x)$ from (42) into the HJB equation and rearrange terms yields:

$$V_1(W) = \max_x \quad \left(\frac{\phi\gamma\kappa}{2(\kappa-2)}\right)x^2 - \left(\frac{\phi\gamma^2}{\kappa-2}\right)x^2 V_1'(W) - W - \frac{\delta}{\lambda}x^\kappa\theta_{\min}^{-\kappa}. \tag{107}$$

Suppose the IC constraint is slack, then the first order condition implies

$$\frac{\phi\gamma}{\kappa-2}\left[\kappa - 2\gamma V_1'(W)\right]x = \frac{\delta}{\lambda}\kappa x^{\kappa-1}\theta_{\min}^{-\kappa}. \tag{108}$$

The solution is

$$x(W) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right)V_1'(W)\right)\left(\frac{\lambda\phi\gamma\theta_{\min}^\kappa}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}}. \tag{109}$$

If (17) is binding, then

$$\lambda \left(\frac{\theta_{\min}}{x}\right)^{\kappa} U(x) = \frac{\lambda \theta_{\min}^{\kappa}}{x^{\kappa}} \left(\frac{\phi \gamma^2 x^2}{\kappa - 2}\right) = \rho, \tag{110}$$

which yields the solution

$$x = \bar{x} \equiv \left[\frac{\lambda \phi \gamma^2 \theta_{\min}^{\kappa}}{\rho(\kappa - 2)}\right]^{\frac{1}{\kappa-2}}. \tag{111}$$

Substituting (72) into (70) implies that there exists $\overline{W}$ such that

$$x(\overline{W}) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'((\overline{W})\right) \left(\frac{\lambda \phi \gamma \theta_{\min}^{\kappa}}{\delta(\kappa - 2)}\right)\right]^{\frac{1}{\kappa-2}} = \bar{x}. \tag{112}$$

That is, $\overline{W}$ solves

$$V_1'(\overline{W}) = \frac{\kappa}{2} \left(\frac{1}{\gamma} - \frac{\delta}{\rho}\right). \tag{113}$$

Meanwhile, substituting $x = \theta_{\min}$ into (70) implies that there exists $\underline{W}$ such that

$$\theta_{\min} = x(W) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(\underline{W})\right) \left(\frac{\lambda \phi \gamma \theta_{\min}^{\kappa}}{\delta(\kappa - 2)}\right)\right]^{\frac{1}{\kappa-2}}. \tag{114}$$

That is, $\underline{W}$ solves

$$V_1'(\underline{W}) = \left(1 - \frac{\delta(\kappa - 2)}{\lambda \phi \gamma \theta_{\min}^2}\right) \frac{\kappa}{2\gamma}. \tag{115}$$

The existence of both $\{\underline{W}, \overline{W}\}$ requires that $\overline{W} > \underline{W}$, which is equivalent to

$$\frac{\kappa}{2} \left(\frac{1}{\gamma} - \frac{\delta}{\rho}\right) < \left(1 - \frac{\delta(\kappa - 2)}{\lambda \phi \gamma \theta_{\min}^2}\right) \frac{\kappa}{2\gamma} \tag{116}$$

or

$$\lambda \phi \theta_{\min}^2 > \frac{\rho(\kappa - 2)}{\gamma^2}$$

while (27) ensures $V_1'(\overline{W}) > 0$.

Proof of Proposition 5: When only a single target is available, by the martingale representation theorem, there exists β_t such that

$$dW_t = \beta_t(dN_t - \lambda dt). \tag{117}$$

Incentive compatibility requires that $\lambda \beta_t \geq \rho$, or $\beta_t \geq \rho/\lambda$. Applying Ito's lemma on (117) implies that the investors' value function solves the HJB equation

$$0 = \max_{\beta \geq \rho/\lambda} \quad -\delta - \lambda\beta V_1'(W) + \lambda(K - \beta - W - V_1(W)) \tag{118}$$

subject to boundary conditions $V_1(0) = 0$. Differentiating (118) with respect to β yields

$$-\lambda(V_1'(W) + 1) \leq 0. \tag{119}$$

Therefore it is optimal to always set $\beta_t = \rho/\lambda$, i.e., the IC constraint is always binding. Replacing β in (118) with ρ/λ and use the definition $V_2 = K - \rho/\lambda$ yields

$$0 = -\delta - \rho V_1'(W) + \lambda(V_2 - W - V_1(W)) \tag{120}$$

as the HJB equation (45).

Proof of Proposition 6: Equations (4) and (23) imply the following comparative statics regarding x^{FB} and $x(\overline{W})$.

$$\frac{\partial x^{FB}}{\partial \lambda} > 0, \quad \frac{\partial x(\overline{W})}{\partial \lambda} > 0, \tag{121}$$
$$\frac{\partial x^{FB}}{\partial \rho} = 0, \quad \frac{\partial x(\overline{W})}{\partial \rho} < 0, \tag{122}$$
$$\frac{\partial x^{FB}}{\partial \delta} < 0, \quad \frac{\partial x(\overline{W})}{\partial \delta} = 0. \tag{123}$$

The first and third lines of these results imply that

$$\frac{\partial}{\partial \lambda}\left(\frac{x(\underline{W})}{x^{FB}}\right) < 0, \tag{124}$$
$$\frac{\partial}{\partial \delta}\left(\frac{x(\underline{W})}{x^{FB}}\right) > 0, \tag{125}$$

given that $x(\underline{W}) = \theta_{\min}$ is a constant, while the second and the third lines imply that

$$\frac{\partial}{\partial \rho}\left(\frac{x(\overline{W})}{x^{FB}}\right) < 0, \tag{126}$$
$$\frac{\partial}{\partial \delta}\left(\frac{x(\overline{W})}{x^{FB}}\right) > 0. \tag{127}$$

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